Bounded excursion stable gravastars and black holes

P. Rocha [1], A.Y. Miguelote [2], R. Chan [2], M.F. da Silva [3], N.O. Santos [4,5,6], and Anzhong Wang [3,4,7*]

[1] *Instituto de Física, Universidade Federal Fluminense, Av. Litorânea, s/n, Boa Viagem 24210-340, Niterói, RJ, Brazil*

[2] *Coordenação de Astronomia e Astrofísica, Observatório Nacional, Rua General José Cristino, 77, São Cristóvão 20921-400, Rio de Janeiro, RJ, Brazil*

[3] *Departamento de Física Teórica, Instituto de Física, Universidade do Estado do Rio de Janeiro, Rua São Francisco Xavier 524, Maracanã 20550-900, Rio de Janeiro - RJ, Brasil*

[4] *LERMA/CNRS-FRE 2460, Université Pierre et Marie Curie, ERGA, Boîte 142, 4 Place Jussieu, 75005 Paris Cedex 05, France*

[5] *School of Mathematical Sciences, Queen Mary, University of London, London E1 4NS, UK*

[6] *Laboratório Nacional de Computação Científica, 25651-070 Petrópolis RJ, Brazil*

[7] *GCAP-CASPER, Department of Physics, Baylor University, Waco, TX 76798, USA*

Dynamical models of prototype gravastars were constructed in order to study their stability. The models are the Visser-Wiltshire three-layer gravastars, in which an infinitely thin spherical shell of stiff fluid divides the whole spacetime into two regions, where the internal region is de Sitter, and the external is Schwarzschild. It is found that in some cases the models represent the "bounded excursion" stable gravastars, where the thin shell is oscillating between two finite radii, while in other cases they collapse until the formation of black holes. In the phase space, the region for the "bounded excursion" gravastars is very small in comparison to that of black holes, but not empty. Therefore, although the existence of gravastars cannot be excluded from such dynamical models, our results do indicate that, even if gravastars indeed exist, they do not exclude the existence of black holes.

I. INTRODUCTION

Black holes are well known and accepted objects not only on the scientific community but also on the general public. This is, for example, attested to partially by the phenomenal success of the well-known film, *Black Holes* (New River Media, 1998), directed by Pappi Corsicato, and by several best selling popular books, such as Kip S. Thorne, *Black Holes and Time Warps: Einstein's Outrageous Legacy* (W.W. Norton & Company, 1995), and Stephen Hawking, *A Brief History of Time* (Bantam Books publisher, 10th edition, 1998). However, the real detection of black holes highly demands their detailed explorations, including the forms of gravitational waves emitted by them. In principle, such a detection can never be conclusive and is fundamentally impossible [1], although there are many strong astronomical evidences for their existence.

The simplest example of black holes is the spherically symmetric vacuum solution of the Einstein field equations,

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{1.1}$$

where the function $f(r)$ is given by

$$f(r) = 1 - \frac{2M}{r}, \tag{1.2}$$

in the units where $c = 1 = G$. The metric is singular at both $r = 0$ and $r = r_g \equiv 2M$. However, the nature of these singularities is different. In particular, the one at $r = 0$ is generic and the spacetime curvature diverges there, while the one at $r = r_g$ is a coordinate one and can be made disappear after proper coordinate transformations. A classical point test particle will freely fall through $r = r_g$ without experiencing anything special. Such a surface is usually called as an event horizon. The studies of the properties of such horizons are fundamentally important, and it is found that there may exist a deep connection between gravity and thermodynamics [2]. The discovery of the quantum Hawking radiation [3] and the black hole entropy which is proportional to the area of the event horizon of the black hole [4] further supports this idea. This interesting relation was first manifested when Jacobson derived the Einstein field equations from the first law of thermodynamics (FLT) by assuming the proportionality of the entropy and the horizon area for all local acceleration horizons [5].

*Electronic address: anzhong˙wang@baylor.edu

For static spherically symmetric and stationary axisymmetric space-times, Padmanabhan *et al* showed that Einstein's equations at the horizon give rise to the FLT [6, 7]. Such considerations were further generalized to the Lovelock gravity [8]. In [9], on the other hand, the gravitational field equations (GFE's) for the nonlinear $f(R)$ theory were derived from the FLT by adding some non-equilibrium corrections.

For the de Sitter space there also exist Hawking temperature and entropy associated with the cosmological event horizons and its thermodynamic laws [10]. In this space, the event horizons coincides with the apparent horizon. For more general cosmological models, event horizons may not exist, but apparent horizons always do, so it is possible to have Hawking temperature and entropy associated with apparent horizons. Along this line, the connection between the FLT of apparent horizons and the Friedmann equations in Einstein's theory with/without the Gauss-Bonnet term, as well as in the Lovelock theory, were found [11]. More recently, it was found that this is also true for the braneworld cosmology [12]. In [13], with the help of a new mass-like function, it was shown that the FLT of apparent horizons in equilibrium can be derived from the Friedmann equations in various theories of gravity, including the Einstein, scalar-tensor, nonlinear $f(R)$, and Lovelock.

Despite of all these theoretical and observational successes, a number of paradoxical problems reparging to black holes also exist [14], which frequently motivate authors to look for other alternatives, in which the endpoints of gravitational collapse are massive stars without horizons. Example of such models include gravastars [15], Bose superfluid [16], and black stars [17], to name only few of them. Among these models, gravastars have received particular attention recently [18], partially due to the tight connection between the cosmological constant and a currently accelerating universe [19], although very strict observational constraints on the existence of such stars may exist [20]. In the original model of Mazur and Mottola (MM) [15], gravastars consist of five layers: an internal core $0 < r < r_1$, described by the de Sitter universe, an intermediate thin layer of stiff fluid $r_1 < r < r_2$, an external region $r > r_2$, described by the Schwarzschild solution (1.1) - (1.2), and two infinitely thin shells, appearing, respectively, on the hypersurfaces $r = r_1$ and $r = r_2$. By properly choosing the free parameters involved, one can show that the two shells can have only tensions but with opposite signs [15]. Visser and Wiltshire (VW) argued that such five-layer models can be simplified to three-layer ones [21], in which the two infinitely thin shells and the intermediate region are replaced by one infinitely thin shell, so that the

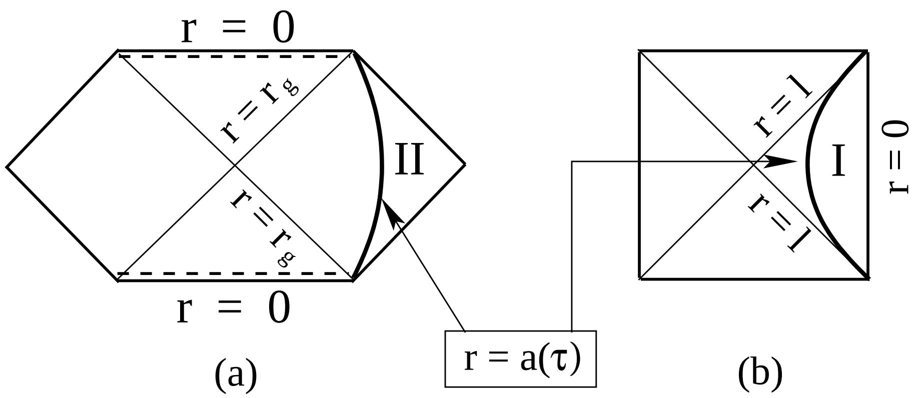

FIG. 1: (a) The Penrose diagram for the Schwarzschild vacuum solution. Region II is the region where $r > a(\tau)$. (b) The Penrose diagram for the de Sitter solution. Region I is the region where $r < a(\tau)$. An infinitely thin shell located at $r = a(\tau)$ connects these two regions to form a dynamical spacetime of a prototype gravastar, as shown in Fig. 2.

function $f(r)$ in the metric (1.1) is given by

$$f(r) = \begin{cases} 1 - \frac{2M}{r}, & r > a(\tau), \\ 1 - \left(\frac{r}{l}\right)^2, & r < a(\tau), \end{cases} \tag{1.3}$$

where $r = a(\tau)$ is a timelike hypersurface, at which the infinitely thin shell is located, and τ denotes the proper time of the thin shell. The constant $l \equiv \sqrt{3/\Lambda}$ denotes the de Sitter radius. On the hypersurface $r = a(\tau)$ Israel junction conditions yield

$$\frac{1}{2}\dot{a}^2 + V(a) = 0, \tag{1.4}$$

where an overdot denotes the derivative with respect to the proper time τ of the thin shell. Therefore, in the region $r > a(\tau)$ the spacetime is locally Schwarzschild, while in the region $r < a(\tau)$ it is locally de Sitter. These two different regions are connected through a dynamical infinitely thin shell located at $r = a(\tau)$ to form a new spacetime of gravastar [cf. Figs. 1 and 2].

To study the dynamics of Eq.(1.4), one can follow two different approaches: one is to prescribe a potential $V(a)$ and leave the equation of state of the shell as derived, and the other is to prescribe an equation of state of the shell and leave the potential $V(a)$ as derived. VW followed the first approach, and studied in details the case where

$$V(a_0) = 0, \quad V'(a_0) = 0, \quad V''(a_0) > 0, \tag{1.5}$$

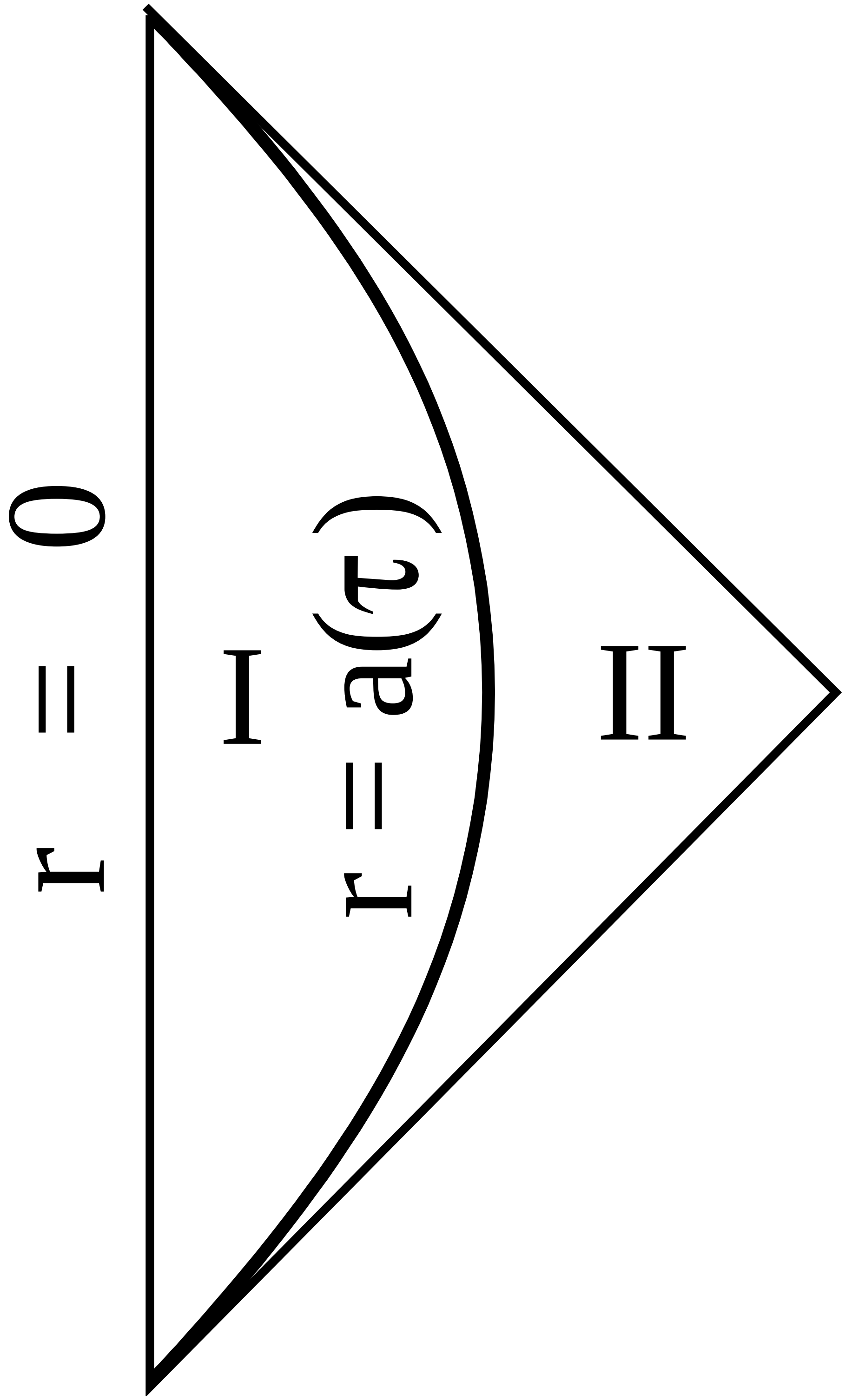

FIG. 2: The Penrose diagram for the solution given by Eq.(1.3). The infinitely thin shell at $r = a(\tau)$ connects the two regions I and II shown in Fig. 1 to form a spacetime of a prototype gravastar.

where a prime denotes the ordinary differentiation with respect to the indicated argument. If and only if there exists such an a_0 for which the above conditions are satisfied, the model is said to be stable. Among other things, VW found that there are many equations of state for which the gravastar configurations are stable, while others are not [21]. Carter studied the same problem and found new equations of state for which the gravastar is stable [22], while De Benedictis *et al* [23] and Chirenti and Rezzolla [24] investigated the stability of the original model of Mazur and Mottola agaist axial-perturbations, and found that gravastars are stable to these perturbations. Chirenti and Rezzolla also showed that their quasi-normal modes differ from those of a black hole of the same mass, and thus can be used to discern a gravastar from a black hole.

As VW noticed, there is a less stringent notion of stability, the so-called "bounded excursion" models, in which there exist two radii a_1 and a_2 such that

$$V\left(a_1\right)=0, \quad V'\left(a_1\right)\leq 0, \quad V\left(a_2\right)=0, \quad V'\left(a_2\right)\geq 0, \tag{1.6}$$

with $V(a) < 0$ for $a \in (a_1, a_2)$, where $a_2 > a_1$.

In this paper, our purpose is twofold: (a) First, we construct three-layer VW dynamical models, some of which represent "bounded excursion" stable gravastars, and some represent the collapse of a prototype gravastar, where the final fate of the collapse is the formation of a black hole. (2) Second, in the phase space we compare the region of such stable gravastars with the one of black holes, and show explicitly that both of them are non-zero, although the former is much smaller than the latter. The rest of the paper is organized as follows: In Sec. II we shall study various cases, in which all the possibilities of forming black holes, gravastars, de Sitter, and Minkowski spacetime exist. In Sec. III we present our main conclusions.

Before turn to the next section, we note some relevant work. In particular, the gravitational collapse of dark energy in the background of dark matter was studied in [25], and it was found that when only dark energy is present, black holes are never formed. When both of them are present, black holes can be formed, due to the condensation of the dark matter. Similar results were obtained in [26–28]. Recently, such studies were further generalized to a homogeneous and isotropic expanding Friedmann-Robertson-Walker universe dominated by dark energy [29].

II. FORMATION OF GRAVASTARS FROM GRAVITATIONAL COLLAPSE

To keep the ideas of MM as much as possible, we consider the thin shell as consisting of a stiff fluid, $\sigma = -\vartheta$, where σ and ϑ denote, respectively, the surface energy density and tension of the shell. Then, we find that [21]

$$\sigma = \sigma_0 \left(\frac{a_0}{a}\right)^4, \tag{2.1}$$

where σ_0 and a_0 are integration constants, and have dimensions of surface energy density and length, respectively. It can be shown that the potential appearing in Eq.(1.4) now can be cast in the form,

$$V(R) = -\frac{1}{2}\left(-1 + \frac{m}{R} + \frac{1}{4R^6} + m^2R^4 + \frac{R^2}{2L^2} - \frac{mR^7}{L^2} + \frac{R^{10}}{4L^4}\right), \tag{2.2}$$

where

$$m \equiv \frac{M}{k^{1/3}}, \quad R \equiv \frac{a}{k^{1/3}}, \quad L \equiv \frac{l}{k^{1/3}}, \tag{2.3}$$

with $k \equiv 4\pi a_0^4 \sigma_0$. Therefore, for any given constants m and L, Eq.(1.4) uniquely determines the collapse of the prototype gravastar. Depending on the initial value R_0, the collapse can form either a black hole, or gravastar, or a Minkowski, or a de Sitter space. In the last case, the thin shell first collapses to a finite non-zero minimal radius and then expends to infinity. To guarantee that initially the spacetime does not have any kind of horizons, cosmological or event, we must restrict R_0 to the range,

$$2m < R_0 < L, \tag{2.4}$$

correspondingly $r_0 \in (r_g, l)$. When $m = 0 = \Lambda$, the thin shell disappears, and the whole spacetime is Minkowski. So, in the following we shall not consider this case, and begin with the one $m = 0$ and $\Lambda \neq 0$.

A. $m = 0$ and $\Lambda \neq 0$

In this case, the spacetime outside the thin shell is flat, and the mass of the shell completely screens the mass of the internal de Sitter spacetime. From Eq.(2.2) we find that

$$V(R) = \frac{1}{2}\left(1 - \frac{1}{4R^6} - \frac{R^2}{2L^2} - \frac{R^{10}}{4L^4}\right). \tag{2.5}$$

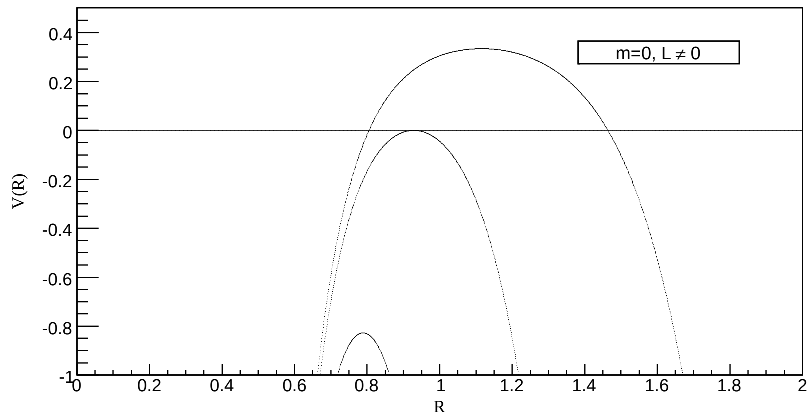

FIG. 3: The potential $V(R)$ for $m = 0$. The top line is for $L = 2.0 > L_c \simeq 0.9588$, the middle line is for $L = L_c$, and the bottom line is for $L = 0.5 < L_c$.

Then, it can be shown that the equations $V(R) = 0$ and $V'(R) = 0$ have the explicit solution,

$$L = L_c \equiv \left[\frac{5}{3}\left(\frac{4}{5}\right)^{8/3}\right]^{1/2} \simeq 0.9588, \quad R = R_{min} \equiv \left(\frac{4}{5}\right)^{1/3}. \tag{2.6}$$

For $L < L_c$ the potential $V(R)$ is strictly negative as shown in Fig. 3. As a result, if the star starts to collapse at $R = R_0$, it will collapse continuously until $R = 0$, whereby a Minkowski spacetime is formed, as shown by the bottom line in Fig. 4. When $L = L_c \simeq 0.9588$, since $R_0 < L_c$, we can see that, similar to the last case, the star will collapse until the center $R = 0$ and turns the whole spacetime into Minkowski, as shown by the middle line in Fig. 4. For $L > L_c$, the potential $V(R)$ is positive between R_1 and R_2, where $R_{1,2}$ are the two real roots of the equation $V(R, L > L_c) = 0$ with $R_2 > R_1 > 0$. In this case, if the star starts to collapse with $R_0 < R_1$, as can be seen from Fig. 3, it will collapse to $R = 0$, whereby a Minkowski spacetime is finally formed. If it starts to collapse with $R_0 > R_2$, it will first collapse to $R = R_2$ and then starts to expand until $R = \infty$, and the whole spacetime is finally de Sitter, as shown by the top line in Fig. 4.

B. $\Lambda = 0$ and $m \neq 0$

In this case, Eq.(2.2) yields,

$$V(R) = \frac{1}{2}\left(1 - \frac{m}{R} - \frac{1}{4R^6} - m^2R^4\right), \tag{2.7}$$

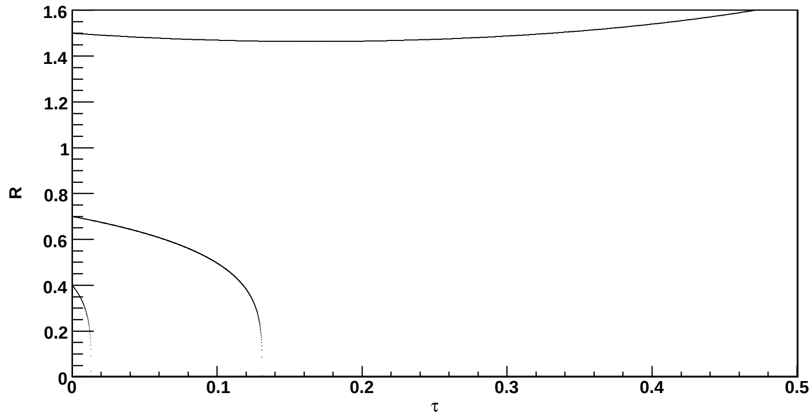

FIG. 4: The motion of the shell $R(\tau)$ vs the proper time, τ, of the shell for $m = 0$. The top line is for $L = 2.0 > L_c \simeq 0.9588$, the middle line is for $L = L_c$, and the bottom line is for $L = 0.5 < L_c$.

from which we find that the equations $V(R) = 0$ and $V'(R) = 0$ have the explicit solution,

$$m = m_c \equiv \frac{3}{4}\left(\frac{4}{5}\right)^{5/3} \simeq 10^{-0.286}, \quad R = R_c \equiv \left(\frac{3}{4m_c}\right)^{1/5} = \left(\frac{5}{4}\right)^{1/3}. \tag{2.8}$$

For $m > m_c$ the potential $V(R)$ is strictly negative as shown in Fig. 5. Then, the collapse always forms black holes, as shown clearly by the bottom line in Fig. 6. For $m = m_c$, there are two different possibilities, depending on the choice of the initial radius R_0. In particular, if the star begins to collapse with $R_0 > R_c$, the collapse will asymptotically approach the minimal radius R_c. Once it collapses to this point, the shell will stop collapsing and remains there for ever, as can be seen from the middle line in Fig. 6. However, in this case this point is unstable, and any small perturbations will lead the star either to expand for ever and leave behand a flat spacetime, or to collapse until $R = 0$, whereby a Schwarzschild black hole is finally formed. On the other hand, if the star begins to collapse with $2m_c < R_0 < R_c$, as shown by Fig. 5, the star will collapse until a black hole is formed. For $m < m_c$, the potential $V(R)$ has a positive maximal, and the equation $V(R, m < m_c) = 0$ has two positive roots $R_{1,2}$ with $R_2 > R_1 > 0$. As in the last case, now there are also two possibilities, depending on the choice of the initial radius R_0. If $R_0 > R_2$, the star will first collapse to its minimal radius $R = R_2$ and then expand to infinity, whereby a Minkowski spacetime is finally formed, as shown by the top line in Fig. 6. If $2m < R_0 < R_1$, the star will collapse continuously until $R = 0$, and a black hole will be finally formed.

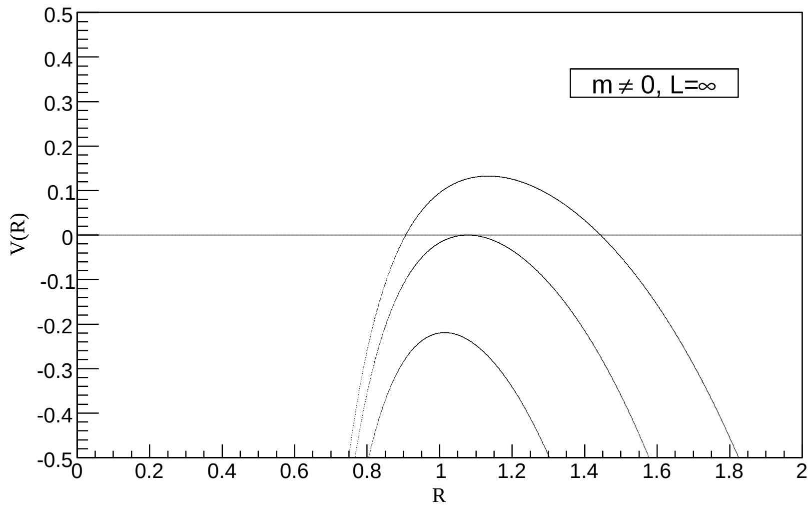

FIG. 5: The potential $V(R)$ for $\Lambda = 0$ (or $L = \infty$). The top line is for $m < m_c \simeq 10^{-0.286}$, the middle line is for $m = m_c$, and the bottom line is for $m > m_c$.

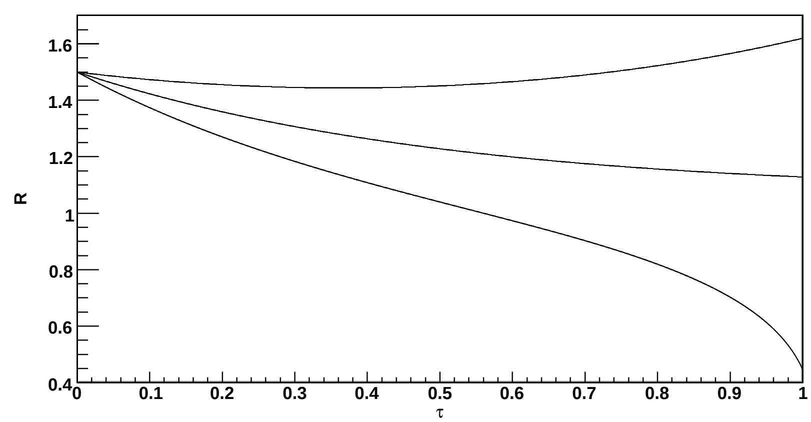

FIG. 6: The motion of the shell $R(\tau)$ vs its proper time τ for $\Lambda = 0$. The top line is for $m < m_c \simeq 10^{-0.286}$, the middle line is for $m = m_c$, and the bottom line is for $m > m_c$.

C. $m \neq 0$ and $\Lambda \neq 0$

In this case, from Eq.(2.2) we find that the equations $V(R) = 0$ and $V'(R) = 0$ have the solution of the form, $m = m_c(L)$ for any given L. The exact dependence of m_c on L cannot be given explicitly. Instead, in the following we consider some representative cases.

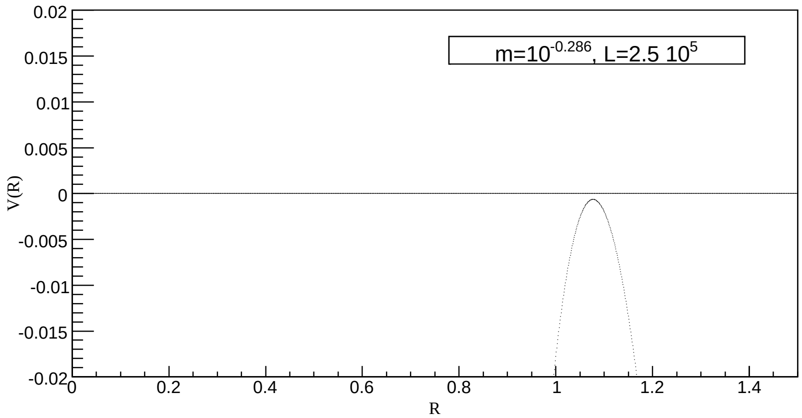

FIG. 7: The potential $V(R)$ for $m = 10^{-0.286}$ and $L = 2.5 \times 10^5$, which is always negative.

1. $m = 10^{-0.286}$, $L = 2.5 \times 10^5$

If we set $m = m_c \simeq 10^{-0.286}$ and graduately turn on the cosmological constant, we find that the potential $V(R)$ becomes completely negative, as shown by Fig. 7. Then, for any given R_0 with $2m < R_0 < L$, the star will always collapse to form a black hole. Comparing it with the case $m = m_c$ and $\Lambda = 0$, we find that the presence of the cosmological constant makes the collapse more like to form black holes than gravastars, or any of the others.

2. $m = 10^{-4}$, $L = 2.5 \times 10^5$

If we keep L fixed, i.e., $L = 2.5 \times 10^5$, and tun m downward, we find that, for $m = 10^{-4}$, the potential takes the shape given by Fig. 8, from which we can see that $V(R) = 0$ now has four real roots, say, R_i, where $R_{i+1} > R_i$. If we choose $R_0 > R_4$, then the star will first collapse to $R = R_4$, and then expand to infinity, whereby a de Sitter space is finally formed. However, if we choose $R_2 < R_0 < R_3$, the collapse will bounce back and forth between $R = R_2$ and $R = R_3$. Such a possibility is shown in Fig. 9. This is exactly the so-called "bounded excursion" model mentioned in [21], but was not studied there or somewhere else. Of course, in a realistic situation, the star will emit both gravitational waves and particles, and the potential shall be self-adjusted to produce a minimum at $R = R_{static}$ where $V\left(R = R_{static}\right) = 0 = V'\left(R = R_{static}\right)$ [cf. Fig. 10], whereby a gravastar is finally formed [21].

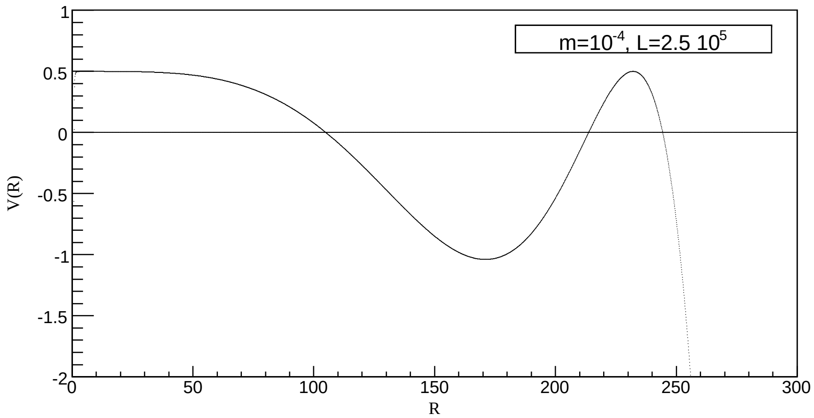

FIG. 8: The potential $V(R)$ for $m = 10^{-4}$ and $L = 2.5 \times 10^5$.

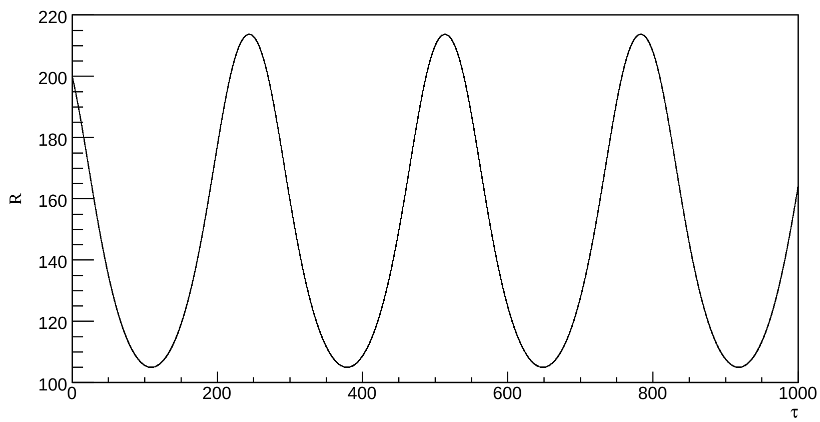

FIG. 9: The development of $R(\tau)$ vs the proper time τ for $m = 10^{-4}$ and $L = 2.5 \times 10^5$.

Although such a shape of potential is very difficult to find in the phase space of m and L, its measurement is not zero. So, one cannot completely exclude the existence of gravastars. However, we do find that it is easy to find potentials that lead to the formation of black holes. Figs. 11 and 12 are for $m = 1.0$ and $L = 3.0$, and $m = 10^{-4}$ and $L = 3.0$, respectively. From these figures we can see that, by properly choosing the initial radius R_0, the collapse always forms black holes. In contrary, in these cases gravastars cannot be formed no matter how to tun R_0. On the other hand, we are not able to find values of m and L for which only gravastars can be formed.

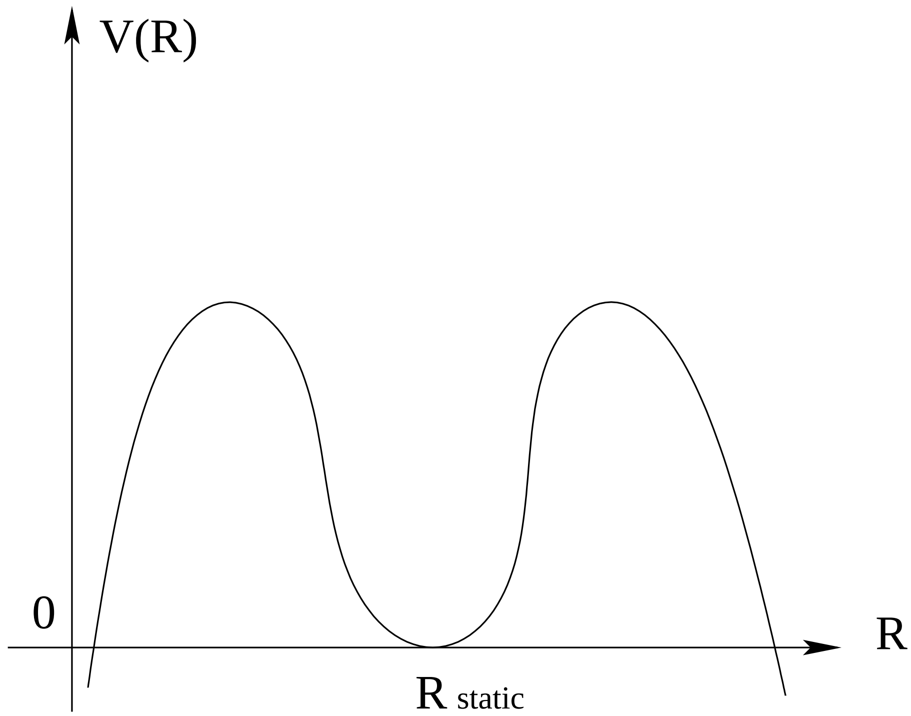

FIG. 10: The potential $V(R)$ for the formation of a gravastar in a realistic collapse, after the star settles down to the minimus point $R = R_{static}$, where $V\left(R = R_{static}\right) = 0 = V'\left(R = R_{static}\right)$.

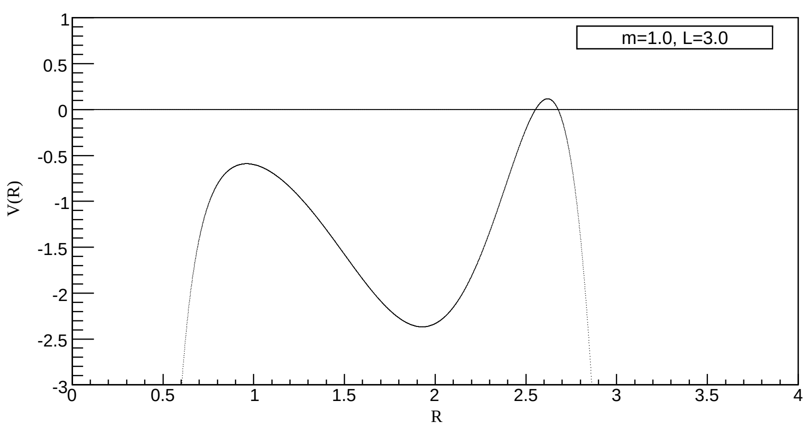

FIG. 11: The potential $V(R)$ for $m = 1.0$ and $L = 3.0$.

III. CONCLUSIONS

In this paper, we have studied the problem of the stability of gravastars by constructing dynamical three-layer models of VW [21], which consists of an internal de Sitter space, a dynamical infinitely thin shell of stiff fluid, and an external Schwarzschild spacetime. We

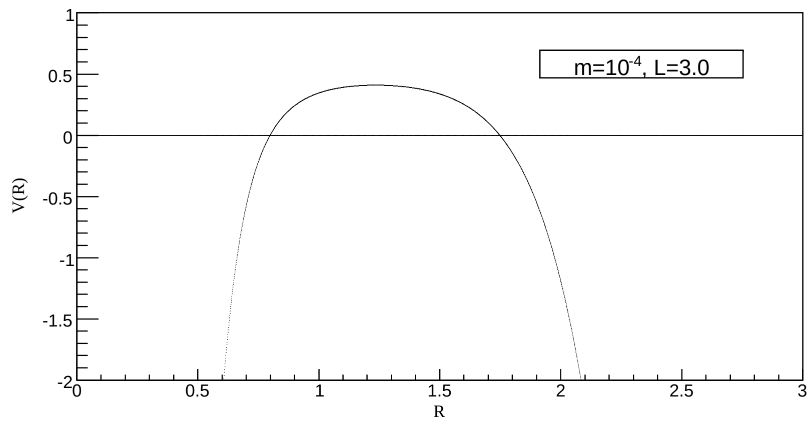

FIG. 12: The potential $V(R)$ for $m = 10^{-4}$ and $L = 3.0$.

have shown explicitly that the final output can be a black hole, a "bounded excursion" stable gravastar, a Minkowski, or a de Sitter spacetime, depending on the total mass m of the system, the cosmological constant Λ, and the initial position R_0 of the dynamical shell. All these possibilities have non-zero measurements in the phase space of m, Λ and R_0, although the region of gravastars is very small im comparing with that of black holes. Therefore, even though the existence of gravastars cannot be completely excluded in these dynamical models, our results do indicate that, even if gravastars exist, they do not exclude the existence of black holes.

Acknowledgments

Part of the work was done when the authors NOS and AW were visiting LERMA/CNRS-FRE. They would like to express their gratitude to the Laboratory for hospitality. This work was partially supported by CNPq (PR, RC, MFS,NOS and AW), FAPERJ (AM), and NSFC under grant No. 10775119 (AW).

[1] M.A. Abramowicz, W. Kluźniak, and J.-P. Lasota, Astron. & Astrophys. **396**, L31-34 (2002).

[2] J.M. Bardeen, B. Carter and S.W. Hawking, Commun. Math. Phys. **31**, 161 (1973).

[3] S.W. Hawking, Commun. Math. Phys. **43**, 199 (1975); **46**, 206(E) (1976).

[4] J.D. Bekenstein, Phys. Rev. D**7**, 2333 (1973).

[5] T. Jacobson, Phys. Rev. Lett. **75**, 1260 (1995).

[6] T. Padmanabhan, Class. Quantum Grav. **19**, 5387 (2002).

[7] D. Kothawala, S. Sarkar and T. Padmanabhan, gr-qc/0701002.

[8] A. Paranjape, S. Sarkar and T. Padmanabhan, Phys. Rev. D**74**, 104015 (2006).

[9] C. Eling, R. Guedens and T. Jacobson, Phys. Rev. Lett. **96**, 121301 (2006).

[10] G. Gibbons and S.W. Hawking, Phys. Rev. D**15**, 2738; 2732 (1977).

[11] R.G. Cai and S.P. Kim, J. High Energy Phys. **02**, 050 (2005).

[12] X.-H. Ge, Phys. Lett. B**651**, 49 (2007).

[13] Y.-G. Gong and A. Wang, Phys. Rev. Lett. **99**, 211301 (2007).

[14] R. M. Wald, Living Rev. Rel. **4**, 6 (2001) [arXiv:gr-qc/9912119].

[15] P.O. Mazur and E. Mottola, "*Gravitational Condensate Stars: An Alternative to Black Holes*," arXiv:gr-qc/0109035; Proc. Nat. Acad. Sci. **101**, 9545 (2004) [arXiv:gr-qc/0407075].

[16] G. Chapline, E. Hohlfeld, R.B. Laughlin, and D.I. Santiago, Int. J. Mod. Phys. A**18**, 3587 (2003) [arXiv:gr-qc/0012094].

[17] T. Vachaspati, "*Black Stars and Gamma Ray Bursts*," arXiv:0706.1203.

[18] D. Horvat and S. Ilijic, arXiv:0707.1636; P. Marecki, arXiv:gr-qc/0612178; F.S.N. Lobo, Phys. Rev. D**75**, 024023 (2007); arXiv:gr-qc/0612030; Class. Quantum Grav. **23**, 1525 (2006); F.S.N. Lobo, Aaron V. B. Arellano, *ibid.*, **24**, 1069 (2007); T. Faber, arXiv:gr-qc/0607029; C. Cattoen, arXiv:gr-qc/0606011; O.B. Zaslavskii, Phys. Lett. B**634**, 111 (2006); C. Cattoen, T. Faber, and M. Visser, Class. Quantum Grav. **22**, 4189 (2005).

[19] E.J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D**15**, 1753 (2006); T. Padmanabhan, arXiv:0705.2533.

[20] A.E. Broderick and R. Narayan, Class. Quantum Grav. **24**, 659 (2007) [arXiv:gr-qc/0701154].

[21] M. Visser and D.L. Wiltshire, Class. Quantum Grav. **21**, 1135 (2004)[arXiv:gr-qc/0310107].

[22] B.M.N. Carter, Class. Quantum Grav. **22**, 4551 (2005) [arXiv:gr-qc/0509087].

[23] A. DeBenedictis, *et al*, Class. Quantum Grav. **23**, 2303 (2006) [arXiv:gr-qc/0511097].

[24] C.B.M.H. Chirenti and L. Rezzolla, arXiv:0706.1513.

[25] R.-G. Cai and A. Wang, Phys. Rev. D**73**, 063005 (2006)[arXiv:astro-ph/0505136].

[26] S. Nath, S. Chakraborty, and U. Debnath, Int. J. Mod. Phys. D**15**, 1225 (2006) [gr-qc/0512120].

[27] U. Debnath and S. Chakraborty, gr-qc/0601049 (2006).

[28] T. Harada, H. Maeda and B. J. Carr, Phys. Rev. D**74**, 024024 (2006) [astro-ph/0604225]; arXiv:0707.0528; H. Maeda, T. Harada, and B.J. Carr, arXiv:0707.0530.

[29] Z.-H. Li and A. Wang, Mod. Phys. Lett. A**22**, 1663 (2007) [arXiv:astro-ph/0607554].